Kiril Datchev
MA 425/525
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Homework 6

Due November 8th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. The book is `https://archive.org/details/complex-variables-2ed-dover-1999-fisher/page/n23/mode/2up`.

- Section 2.6 (pages 168 and 169) #27, 28 (show this by finding a number M involving e and π such that $|C(z)| \leq M$ when z is on γ_N, regardless of N), 32, 33.
- Nonbook problem: Use the answer to # 33 above with a good choice of w to evaluate $\sum_{k=1}^{\infty} \frac{1}{1+\pi^2 k^2}$. Simplify your answer as much as possible.
- You don't have to do this one but you may like to take a look at # 34. The annoying thing there is navigating branches of log, but note that those issues disappear in the final formula (11), which is a version for $\sin z$ of the polynomial factorization formula $P(z) = a(z - p_1) \cdots (z - p_n)$, where $p_1, \dots p_n$ are the roots of the polynomial.

Hints: For 2.6.28, write $|\cot \pi z| = \frac{|1+e^{ia}e^{b}|}{|1-e^{ia}e^{b}|}$ for suitable real numbers a and b, and use the way that either a or b simplifies on each side of the square. For 2.6.33, simplify $\frac{1}{k(k-w)} + \frac{1}{(-k)(-k-w)}$ and use the result to rewrite the sum over $k \neq 0$ as a sum over $k \geq 1$.