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MA 425/525
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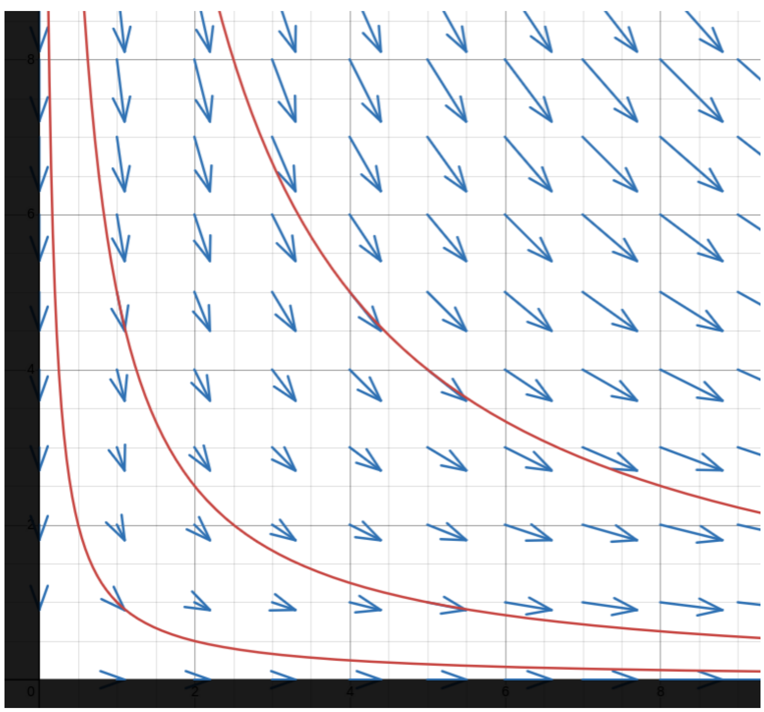
Homework 6

Due November 6th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. The book is `https://archive.org/details/complex-variables-2ed-dover-1999-fisher/page/n23/mode/2up`.

Do 2.6.27, 2.6.29 (for this one you may use without checking it that $|C(z)| \leq 4$ on γ_N) from page 169, 3.1.12, 3.1.15, 3.1.16, 3.1.17 from page 180.

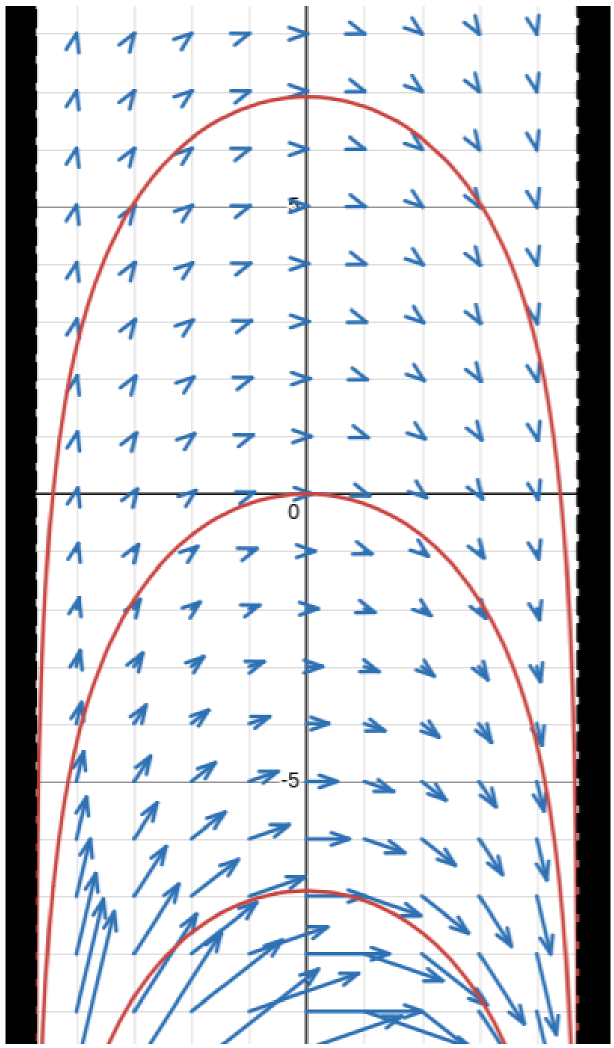
Also do the following:

1. For the vector field $v(z) = \bar{z}$, recall from `https://www.math.purdue.edu/~kdatchev/425|525/lift.pdf` that the complex velocity is obtained by taking the complex conjugate to get $f(z) = z$, and a complex potential is obtained by taking an antiderivative to get $F(z) = z^2/2$. The curves $\operatorname{Im} F(z) = C$ give streamlines of v. This represents smooth flow around a corner: see the picture below; the blue arrows are the vector field, and the red curves are three of the streamlines.



Let $p = 3^{3/4}e^{i\pi/3}$ and let S be the streamline passing through p. Which point on S is closest to the origin?

2. The vector field $v(z) = e^{-i\bar{z}}$ represents a smooth flow in a channel which is at the same speed but in opposite directions on the opposite edges. (The flow speed grows exponentially as you go down the channel, which seems unrealistic but leads to a nice formula for the streamlines.) The picture below is analogous to the one above.



Let $p = -\pi/3 + ib$, where b is a real number, let S be the streamline passing through p, and let ic be the point where S crosses the imaginary axis. Find $c - b$.

Hints: For 2.6.27, to find the poles determine when the denominator is zero by solving $e^{i\pi z} = e^{-i\pi z}$. Then find the order and residue at the pole at $z = 0$, then show that $C(z) = C(z + k)$ for any integer k and use this to deduce the other orders and residues. For 3.1.16, apply Theorem 3 with g replaced by $g - f$. For 1 and 2, use the given point to solve for C in $\operatorname{Im} F(z) = C$.